

Non-partial Reality

Yu Shi

Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel

Study on pre- and postselected quantum system indicates that “product rule” and “sum rule” for elements of reality should be abandoned. We show that this so-called non-partial realism can refute arguments against hidden variables in a unified way, and might save local realism.

PACS numbers: 03.65.Bz

Einstein, Podolsky and Rosen (EPR) showed that quantum mechanical description of physical reality is not complete based on a sufficient condition for physical reality: “If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.” [1,2]. In Bohm’s version [3], consider a pair of spin- $\frac{1}{2}$ particles in a singlet state

$$|\Psi\rangle = \frac{1}{2}(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle), \quad (1)$$

which is an eigenstate of $\hat{\sigma}_{1\hat{n}}\hat{\sigma}_{2\hat{n}}$, with \hat{n} an arbitrary direction. The spin component of particle 2 may be predicted with certainty by measuring that of particle 1 in the same direction, and vice versa. Therefore every spin component of each particle is an element of reality, and thus quantum mechanics is incomplete. EPR believed a complete theory is possible. von Neumann [4], Gleason [5], Jauch and Piron [6] gave proofs of impossibility of a hidden variable theory. Bell pointed out unreasonable postulates in these proofs by noting that noncommuting observables are measured in different experiments [7]. Later he proved the Bell theorem that a local hidden variable theory is inconsistent with quantum mechanics by deriving an inequality from the hidden variables for EPR-Bohm setup, this equality is violated by quantum mechanics [8]. On the other hand, Kochen and Specker (KS) also excluded the possibility of (noncontextual) hidden variables by proving that it is impossible to assign definite value to each of a set of commuting observables [9]. Recently Bell theorem without inequality received much interest. An earlier proof was given by Heywood and Redhead [10] with the aid of KS theorem. A few years ago Greenberger, Horn and Zeilinger gave a proof with EPR-Bell argument for certain states of three or more spin- $\frac{1}{2}$ particles [11], Mermin gave a simplified version [12]. Hady gave proofs for two spin- $\frac{1}{2}$ particles [13], a streamlined version was given by Goldstein [14]. Non-locality of a single photon was also claimed by Hardy [15], while interpreted as multiparticle state by others

[16]. Among recent progress on KS theorem, Peres gave a simple KS argument but relied on singlet state [17], Mermin gave simple general KS arguments for two and three spin- $\frac{1}{2}$ particles, and found the relevance of the latter with Bell theorem [18].

Redhead extended EPR sufficient condition for the element of physical reality to: “If we can predict with certainty, or at any rate with probability one, the result of measuring a physical quantity at time t , then at time t , there exists an element of reality corresponding to this physical quantity and having a value equal to the predicted measurement result” [19]. Redhead condition does not insist on “without in any way disturbing a system” though stresses “at time t ”. Actually every physical process happens at a certain time, so “at time t ” is also implicit in EPR condition. In the gedanken experiments of EPR and Bohm, the Hamiltonians are zero, therefore the elements of reality do not change with time and thus the time is not explicit. Therefore Redhead condition is weaker than EPR condition while the converse is stronger. We will consider both. We also extend “predict” to “infer”, as done by Vaidman for Redhead condition [20].

We will explain that in all the above-mentioned work after EPR, there is an implicit premise underlying the deduction leading to contradiction: the reality is “partial”, i.e., the elements of reality obey a “sum rule” and a “product rule”. However, recently Vaidman showed that the “product rule” should be abandoned [20]. We add that “sum rule” should also be abandoned; actually “sum rule” is a sufficient condition of “product rule”. Therefore all the arguments against hidden variables can be refuted in a unified way. In particular, since “partiality” of reality is a premise in the proofs of Bell theorem, local realism might be saved.

The notations are described. The letter with a hat, like \hat{A} , represents an observable (physical quantity), as well as the corresponding operator in quantum mechanics. The letter without a hat, like A , denotes the result of measuring \hat{A} . The element of reality corresponding to the observable \hat{A} is denoted as $\{\hat{A}\}$, or $\{\hat{A}\}(\lambda)$, where λ is the hidden variable. Therefore if A is obtained with probability equal to 1, then $\{\hat{A}\}(\lambda) = A$; if A is obtained with probability not equal to 1, then it is unknown whether $\{A\}$ exists.

First we briefly review Vaidman’s results. Consider a quantum system which is prepared in a state $|\Psi_1\rangle$ at $t_1 < t$, and is found in state $|\Psi_2\rangle$ at $t_2 > t$. The Hamiltonian is let to be zero for simplicity. Suppose \hat{A} is measured at t . If either $|\Psi_1\rangle$ or $|\Psi_2\rangle$ is an eigenstate

of \hat{A} , the outcome is certainly the corresponding eigenvalue. For a pre- and postselected system it might be that the result of measuring \hat{A} is certain, i.e., with probability equal to 1, even though neither $|\Psi_1\rangle$ nor $|\Psi_2\rangle$ is an eigenstate of \hat{A} . Suppose \hat{B} can also be measured with probability equal to 1. Vaidman showed that the outcome of measuring $\hat{A}\hat{B}$ may be uncertain, and may be certain but needs not equal to the product of results of respective measurements of \hat{A} and \hat{B} . Considering elements of reality defined by both prediction and retrodiction, Vaidman said that Redhead condition continues to hold with “predict” changed to “infer”. while the “product rule” should be abandoned. Using our notations, the result is as follows. The existence of $\{\hat{A}_1\}$ and $\{\hat{A}_2\}$ does not imply the existence of $\{\hat{A}_1\hat{A}_2\} = \{\hat{A}_1\}\{\hat{A}_2\}$. Even if $\{\hat{A}_1\hat{A}_2\}$ exists, it needs not equal $\{\hat{A}_1\}\{\hat{A}_2\}$. Vaidman used his result to refute previous arguments against Lorentz invariance [13,21]. The above result is a starting point of our work. However, we do not agree on his interpretation of the breakdown of “product rule” as that “joint measurements of commuting operators in the considered situations invariably disturb each other”, therefore might be a manifestation of nonlocality [22]. We will come back to this point finally.

Now we extend the breakdown of “product rule” to “sum rule” which asserts that the existence of $\{\hat{A}_1\}$ and $\{\hat{A}_2\}$ implies the existence of $\{r_1\hat{A}_1 + r_2\hat{A}_2\} = r_1\{\hat{A}_1\} + r_2\{\hat{A}_2\}$, where r_1 and r_2 are real numbers.

As done by Vaidman, consider EPR-Bohm setup. At time t_1 , $|\Psi_1\rangle = |\Psi\rangle$ as given by Eq. (1). At time t_2 , $\hat{\sigma}_{1x}$ and $\hat{\sigma}_{2y}$ are measured and certain results are obtained. Suppose the results are $\sigma_{1x} = 1$ and $\sigma_{2y} = 1$, then $|\Psi_2\rangle = |\uparrow_{1x}\uparrow_{2y}\rangle$. If at time t , $t_1 < t < t_2$, a measurement is performed on $\hat{\sigma}_{1y}$, then the outcome that $\sigma_{1y}(t) = -\sigma_{2y}(t_2) = -1$ is certain. If, instead, a measurement is performed on $\hat{\sigma}_{2x}$, the outcome is also certain: $\sigma_{2x}(t) = -\sigma_{1x}(t_2) = -1$. This can be verified by the formula calculating probabilities for the results of an intermediate measurement performed on a pre- and postselected system [23]:

$$p(A = a_n) = \frac{|\langle \Psi_2 | \hat{P}(A = a_n) | \Psi_1 \rangle|^2}{\sum_k |\langle \Psi_2 | \hat{P}(A = a_k) | \Psi_1 \rangle|^2}, \quad (2)$$

where $p(A = a_n)$ is the probability for an intermediate measurement of \hat{A} between $|\Psi_1\rangle$ and $|\Psi_2\rangle$ to yield $A = a_n$, $\hat{P}(A = a_k)$ is the projection operator onto the subspace with eigenvalue a_k . However, Vaidman showed that $p(\sigma_{1y}\sigma_{2x} = 1) = 0$, violating the “product rule”.

Now we calculate $p(\sigma_{1y} + \sigma_{2x} = -2)$. The relevant projection operators are $\hat{P}(\sigma_{1y} + \sigma_{2x} = 2) = |\uparrow_{1y}\uparrow_{2x}\rangle\langle\uparrow_{1y}\uparrow_{2x}|$, $\hat{P}(\sigma_{1y} + \sigma_{2x} = 0) = |\uparrow_{1y}\downarrow_{2x}\rangle\langle\uparrow_{1y}\downarrow_{2x}| + |\downarrow_{1y}\uparrow_{2x}\rangle\langle\downarrow_{1y}\uparrow_{2x}|$, $\hat{P}(\sigma_{1y} + \sigma_{2x} = -2) = |\downarrow_{1y}\downarrow_{2x}\rangle\langle\downarrow_{1y}\downarrow_{2x}|$. Then Eq. (2) yields $p(\sigma_{1y} + \sigma_{2x} = -2) = 1/6$, $p(\sigma_{1y} + \sigma_{2x} = 2) = 1/6$, and $p(\sigma_{1y} + \sigma_{2x} = 0) = 2/3$. Therefore the “sum rule” is also

violated; it does not follow that $\sigma_{1y} + \sigma_{2x}$ is an element of reality. It can be checked that both “product rule” and “sum rule” are violated if $|\Psi_2\rangle$ is any of the nine product states. Note that $\hat{\sigma}_{1y} + \hat{\sigma}_{2x}$ can be measured using local interactions [24].

From the definition of element of reality, the existence of $\{\hat{A}\}$ implies the existence of $\{\hat{A}^2\} = \{\hat{A}\}^2$, since we can infer the result of measuring \hat{A}^2 to be $\{\hat{A}\}^2$ with probability equal to 1 if $\{\hat{A}\}$ exists. Therefore it can be shown as follows that the validity of “sum rule” implies the validity of “product rule”, and thus violation of the latter implies violation of the former. Assuming the “sum rule”, then $\{\hat{A}_1\}^2 + 2\{\hat{A}_1\}\{\hat{A}_2\} + \{\hat{A}_2\}^2 = [\{\hat{A}_1\} + \{\hat{A}_2\}]^2 = \{\hat{A}_1 + \hat{A}_2\}^2 = \{(\hat{A}_1 + \hat{A}_2)^2\} = \{\hat{A}_1^2 + 2\hat{A}_1\hat{A}_2 + \hat{A}_2^2\} = \{\hat{A}_1\}^2 + 2\{\hat{A}_1\hat{A}_2\} + \{\hat{A}_2\}^2$, therefore the “product rule” that $\{\hat{A}_1\hat{A}_2\} = \{\hat{A}_1\}\{\hat{A}_2\}$ is obtained. This deduction is similar to that of Kochen and Specker for the multiplicativity from the additivity of functions of an observable [9].

Therefore, study on pre- and postselected quantum system reminds us that the reality is non-partial. We call the reality “partial” if the “sum rule” and “product rule” are valid, i.e., the existence of elements of reality $\{\hat{A}_1\}$ and $\{\hat{A}_2\}$ implies the existence of elements $\{r_1\hat{A}_1 + r_2\hat{A}_2\}$, which equals $r_1\{\hat{A}_1\} + r_2\{\hat{A}_2\}$, and the existence of $\{\hat{A}_1\hat{A}_2\}$, which equals $\{\hat{A}_1\}\{\hat{A}_2\}$, where r_1 and r_2 are real numbers.

We use the term “partial” since according to Kochen and Specker a set of observable is called a “partial algebra” if it is closed under the formation of “partial operations” of sums and products for pairwise commuting observables in accordance with the following rules: if $\hat{A}_1 = f_1(\hat{B})$ and $\hat{A}_2 = f_2(\hat{B})$, then $r_1\hat{A}_1 + r_2\hat{A}_2 = (r_1f_1 + r_2f_2)(\hat{B})$ and $\hat{A}_1\hat{A}_2 = (f_1f_2)(\hat{B})$. They argued that a necessary condition for the existence of hidden variables is that the partial algebra be imbeddable in a commutative algebra (such as the algebra of all real-valued function on a phase space), and thus be preserved by the functions $f_{\hat{A}}(\lambda) = A$ corresponding to the observable \hat{A} and its eigenvalue A , where λ is the hidden variable. Therefore the existence of a homomorphism of the partial algebra of Hermitian operator into the real set is necessary. Based on the nonexistence of such homomorphism, possibility of (noncontextual) hidden variables was excluded. This was demonstrated by considering the angular momentum operator equation $\hat{J}^2 = \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2$ in $j = 1$ state. Since \hat{J}_x^2 , \hat{J}_y^2 , \hat{J}_z^2 commute among themselves as well as with \hat{J}^2 which yields 2 and are therefore simultaneously measurable, always yielding one 0 and two 1, a hidden variable theory should assign to each direction a definite value of the component of \hat{J}^2 . It was shown that no such assignment exists. In the argument of Heywood and Redhead, EPR argument for a pair of spin-1 particles with total spin zero yields the conclusion that there exists definite value of 0 or 1 for each compo-

ment of \hat{J}^2 , contradicting KS theorem. Now we know that in the pre- and postselected quantum system, the partial algebra is not preserved. Though \hat{A}_1 and \hat{A}_2 commute and can be measured with probability 1, a partial operation of them might not. Even if it can be measured with probability 1, the result might not be the partial operation of the results of A_1 and A_2 . It implies that the reality is not partial, and thus the anticipated hidden variable theory giving the functions $f_{\hat{A}}(\lambda) = A$ does not embed partial algebraic structure. In the example of $j = 1$ state, although an anticipated hidden variable theory gives definite value (an element of reality) of either 0 or 1 to each component of \hat{J}^2 , and definite value of 2 to J^2 , there is no reason to require these elements of reality to obey the same equation of the operators, i.e. the “sum rule” is violated. In the recent simplified forms of Kochen-Specker argument using two or three spin- $\frac{1}{2}$ particles by Peres and by Mermin, instead of “sum rule”, “product rule” was used.

Bell pointed out the irrelevance of von Neuman’s impossibility proof by rejecting the postulate of additivity of expectation value. Gleason theorem reduced the additivity postulate to be only of commuting observables. But Bell continued to show that in the corollary excluding hidden variables, as well as in the version of Jauch and Piron, it was still assumed, in a tacit way, that the measuring result of an observable is independent of measurement made on other observables not commuting with the former. Now we can see that in addition to Bell’s arguments, the postulate of additivity of expectation value even only for commuting observable is unrealistic given the non-partiality of reality.

In deriving Bell theorem in the form of inequality for EPR-Bohm setup, Bell studied the hidden variable theoretic expectation value $P(\hat{n}_1, \hat{n}_2) = \int d\lambda \rho(\lambda) \{\hat{\sigma}_{1\hat{n}_1}\}(\lambda) \{\hat{\sigma}_{2\hat{n}_2}\}(\lambda)$ supposed to correspond to quantum mechanical expectation value $\langle \hat{\sigma}_{1\hat{n}_1} \hat{\sigma}_{2\hat{n}_2} \rangle$. Then as a special case, for the perfect correlation, there must be $\{\hat{\sigma}_{1\hat{n}}\}(\lambda) = -\{\hat{\sigma}_{2\hat{n}}\}(\lambda)$ so that $P(\hat{n}, \hat{n}) = -1$ as required by quantum mechanical result. In his derivation, the “product rule” was used. It was implicitly assumed that corresponding to $\hat{\sigma}_{1\hat{n}_1} \hat{\sigma}_{2\hat{n}_2}$ there must exist element of reality $\{\hat{\sigma}_{1\hat{n}_1} \hat{\sigma}_{2\hat{n}_2}\}$ and it equals to $\{\hat{\sigma}_{1\hat{n}_1}\} \{\hat{\sigma}_{2\hat{n}_2}\}$. Through EPR argument, one can be convinced that there exist elements of reality corresponding to $\hat{\sigma}_{1\hat{n}_1}$ and $\hat{\sigma}_{2\hat{n}_2}$ for arbitrary \hat{n}_1 and \hat{n}_2 , but it does not follow that there is an element of reality corresponding to their product, let alone this element of reality is just equal to $\{\hat{\sigma}_{1\hat{n}_1}\} \{\hat{\sigma}_{2\hat{n}_2}\}$. In another word, $P(\hat{n}_1, \hat{n}_2)$ given above does not correspond to $\langle \hat{\sigma}_{1\hat{n}_1} \hat{\sigma}_{2\hat{n}_2} \rangle$ if the “product rule” is abandoned.

The perfect correlation needs more analyses. In addition to Bell’s method obtaining $\{\hat{\sigma}_{1\hat{n}}\} = -\{\hat{\sigma}_{2\hat{n}}\}$, there might be the following reasoning without considering the general $P(\hat{n}_1, \hat{n}_2)$. Since the singlet state (1) is an eigenstate of $\hat{\sigma}_{1\hat{n}} \hat{\sigma}_{2\hat{n}}$, it is known that $\{\hat{\sigma}_{1\hat{n}} \hat{\sigma}_{2\hat{n}}\} = \sigma_{1\hat{n}} \sigma_{2\hat{n}} = -1$

is an element of reality according to Redhead condition (this might be doubtful if EPR’s “without disturbing” is insisted). Then two alternative ways might follow. (a) Use the “product rule” to obtain $\{\hat{\sigma}_{1\hat{n}}\} \{\hat{\sigma}_{2\hat{n}}\} = \{\sigma_{1\hat{n}} \sigma_{2\hat{n}}\} = -1$. (b) It is convinced by EPR argument that $\{\sigma_{1\hat{n}}\}$ and $\{\sigma_{2\hat{n}}\}$ exist. Hence in the identity $\sigma_{1\hat{n}} \sigma_{2\hat{n}} = -1$, one just replace $\sigma_{1\hat{n}}$ with $\{\sigma_{1\hat{n}}\}$, and $\sigma_{2\hat{n}}$ with $\{\sigma_{2\hat{n}}\}$, or considering $\{\hat{\sigma}_{1\hat{n}}\}$ ($\{\hat{\sigma}_{2\hat{n}}\}$) is inferred by $\sigma_{2\hat{n}}$ ($\sigma_{1\hat{n}}$), replace $\sigma_{1\hat{n}}$ with $-\{\hat{\sigma}_{2\hat{n}}\}$ and $\sigma_{2\hat{n}}$ with $-\{\hat{\sigma}_{1\hat{n}}\}$. But we have to say that this is a circular reasoning: $\{\hat{\sigma}_{1\hat{n}}\} = \sigma_{1\hat{n}} = -\sigma_{2\hat{n}}$ or $\{\hat{\sigma}_{2\hat{n}}\} = \sigma_{2\hat{n}} = -\sigma_{1\hat{n}}$ is just deducted from $\sigma_{1\hat{n}} \sigma_{2\hat{n}} = -1$, one cannot put the result back to get more result; by doing this, it means that it has been certain that the measuring result on another particle, by which the element of reality for one particle is inferred, is also an element of reality in the meantime, but this is just the anticipated result of doing this. From physics viewpoint, particle 2 must be disturbed in order that $\{\hat{\sigma}_{1\hat{n}}\}$ is inferred, and vice versa. The probability of inferring $\{\hat{\sigma}_{1\hat{n}}\}$, which is equal to 1, is actually the conditional probability that $\{\hat{\sigma}_{1\hat{n}}\} = -\sigma_{2\hat{n}}$; because there is no causal connection between these two particles the conditional probability is just equal to the probability of inferring $\{\hat{\sigma}_{1\hat{n}}\}$. But in the meantime, the value of $\sigma_{2\hat{n}}$ cannot be obtained with certainty. Therefore the perfect correlation is not between elements of reality but between the element of reality of one particle and the measuring result of another particle, which is obtained with disturbance and with a probability not equal to 1. In conclusion, $\{\hat{\sigma}_{1\hat{n}} \hat{\sigma}_{2\hat{n}}\} = -1$ cannot be changed to $\{\hat{\sigma}_{1\hat{n}}\} \{\hat{\sigma}_{2\hat{n}}\} = -1$.

In GHZ-Mermin approaches to Bell theorem without inequality, contradiction was found by dealing with perfect correlation. GHZ’s original method is just similar to Bell’s. Consider, e.g., $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1 \uparrow_2 \downarrow_3 \downarrow_4\rangle - |\downarrow_1 \downarrow_2 \uparrow_3 \uparrow_4\rangle)$. Corresponding to $\langle \hat{\sigma}_{1\phi_1} \hat{\sigma}_{2\phi_2} \hat{\sigma}_{2\phi_2} \hat{\sigma}_{2\phi_2} \rangle$, where spins are assumed to be co-planar for simplicity, the “product rule” is assumed to obtain $P = \int d\lambda \rho(\lambda) \{\hat{\sigma}_{1\phi_1}\}(\lambda) \{\hat{\sigma}_{2\phi_2}\}(\lambda) \{\hat{\sigma}_{3\phi_3}\}(\lambda) \{\hat{\sigma}_{4\phi_4}\}(\lambda)$, though this expression was not written out explicitly. Then from quantum mechanical result for perfect correlation, several equations identifying the integrand to 1 or -1 are given and lead to contradiction. In Mermin’s version, consider, e.g., $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1 \uparrow_2 \uparrow_3\rangle - |\downarrow_1 \downarrow_2 \downarrow_3\rangle)$, which is a simultaneous eigenstate of $\hat{\sigma}_{1x} \hat{\sigma}_{2y} \hat{\sigma}_{3y}$, $\hat{\sigma}_{1y} \hat{\sigma}_{2x} \hat{\sigma}_{3y}$, $\hat{\sigma}_{1y} \hat{\sigma}_{2y} \hat{\sigma}_{3x}$, and $\hat{\sigma}_{1x} \hat{\sigma}_{2x} \hat{\sigma}_{3x}$, with eigenvalues 1, 1, 1, and -1 , respectively. In the four identities, each spin operator is replaced by the corresponding element of reality, then inconsistency is obtained. This reasoning is just the circular one analyzed above.

Hardy-Goldstein approaches were also based on “product rule”. It suffices to analyze the streamlined version by Goldstein. Consider $|\Psi\rangle = a|v_1\rangle|v_2\rangle + b_1|u_1\rangle|v_2\rangle + b_2|v_1\rangle|u_2\rangle$, where $ab_1b_2 \neq 0$, $|u_i\rangle$ and $|v_i\rangle$ are basis of particle i . Let $\hat{U}_1 = |u_1\rangle\langle u_1|$ etc., and

$|w_i\rangle \propto a|v_i\rangle + b_i|u_i\rangle$. One has: (i) $U_1 U_2 = 0$. (ii) $U_1 = 0$ implies $W_2 = 1$ while $U_2 = 0$ implies $W_1 = 1$. Therefore assumption of local hidden variables contradict (iii) $W_1 = W_2 = 0$ with nonvanishing probability. In this deduction, perfect correlation (i) implies $\{U_1 U_2\} = 0$, but $\{U_1\}\{U_2\} = 0$ does not follow if the “product rule” is abandoned. Then the implication (ii) from $|\Psi\rangle$ cannot be made; one should consider the collapsed state as follows. Suppose U_2 is measured, if $U_2 = 0$, the state collapses to be $|w_1\rangle$ or $|v_2\rangle$, indeed $W_1 = 1$; but if $U_2 = 1$, the state becomes $c|v_1\rangle + |u_2\rangle$: now it is certain $U_1 = 0$, but there are both non-zero probabilities for W_2 to be 1 and 0, in the meantime W_1 may also be 0 or 1. So there is no contradiction with (iii). Only if $\{U_1\}\{U_2\} = 0$ can Goldstein’s reasoning be valid.

Both Bell Theorem and the simplified form of Kochen-Specker arguments of Peres and Mermin use “product rule”, which is applied to arbitrary observables for the latter, while only to causally disconnected ones for the former. It is also found that the “product rule” is related to the “sum rule”, which was used in original KS theorem. This appears to be the origin of the relation between simple forms of the two Theorems exposed by Mermin in GHZ state.

Finally, we note that non-partiality is not identical with either contextual or nonlocality. Non-partiality means that the element of reality corresponding to the sum or product of two observables is not the same partial operation of the elements of reality corresponding to the two observables, while contextuality concerns the relation between the two independent observables; referring to that the measuring result of one observable depends on the simultaneous measurement made on the other observable, even though they are compatible. Similarly, nonlocality also concerns the relation between two independent observables; in this case they are causally disconnected. Although contextuality or nonlocality gives rise to non-partiality (this is why non-local hidden variable theory, such as that of Bohm [25], can work), non-partiality might be noncontextual and local. In addition to logical reasoning, a support to this possibility comes from that the partial operation of operators of causally disconnected particles might be measured locally [24]. Further clarification on this point is deserved.

To summarize, study on pre- and postselected quantum systems indicates that both “product rule” and “sum rule” for the elements of reality should be abandoned. We expose that all arguments against hidden variables were based on the assumption of partiality of reality, therefore can be refuted unifiedly by non-partial realism, which might thus save local realism. At least, we have reached a unified understanding of Bell theorem and KS theorem.

- [1] A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 770 (1935).
- [2] For a review of Ref. [1,3-9,25], see M. Jammer, *The Philosophy of Quantum Mechanics*, (Wiley-Interscience Publications, NY, 1974), Chapter 6-7.
- [3] D. Bohm, *Quantum Theory*, (Prentice-Hall, Englewood cliffs, NJ, 1951), pp. 614-616.
- [4] J. von Neumann, *Mathematische Grundlagen der Quanten-Mechanik*, (Verlag Julius-Springer, Berlin, 1932); *Mathematical Foundations of Quantum Mechanics*, (Princeton University, Princeton, 1955).
- [5] A. M. Gleason, *J. Math. Mech.* **6**, 885 (1957).
- [6] J. M. Jauch and C. Piron, *Helv. Phys. Acta* **36**, 827 (1963).
- [7] J. S. Bell, *Rev. Mod. Phys.* **38**, 447 (1966).
- [8] J. S. Bell, *Physics* **1**, 195 (1964).
- [9] S. Koch and E. P. Specker, *J. math. Mech.* **17**, 59 (1967).
- [10] P. Heywood and M. I. G. Redhead, *Found. Phys.* **13**, 481 (1983).
- [11] D. M. Greenberger, M. A. Horne, A. Shimony, and A. Zeilinger, *Am. J. Phys.* **58**, 1131 (1990).
- [12] N. D. Mermin, *Phys. Today* **43**(6), 9 (1990); *Am. J. Phys.* **58**, 731 (1990).
- [13] L. Hardy, *Phys. Rev. Lett.* **68**, 2981 (1992); *ibid.* **71**, 1665 (1993).
- [14] S. Goldstein, *Phys. Rev. Lett.* **72**, 1951 (1994).
- [15] L. Hardy, *Phys. Rev. Lett.* **73**, 2279 (1994).
- [16] L. Vaidman, *Phys. Rev. Lett.*, **75**, 2063 (1995); D. M. Greenberger, M. A. Horn, and A. Zeilinger, *ibid.* **75**, 2064 (1995). See also reply of L. Hardy, *ibid.* **75**, 2065 (1995).
- [17] A. Peres, *Phys. Lett. A* **151**, 107 (1990).
- [18] N. D. Mermin, *Phys. Rev. Lett.* **65**, 3373 (1990); Mermin said that unlike GHZ state, KS argument of Peres for the singlet state [17] cannot be casted into the form of Bell-EPR argument, since though there is perfect correlation between $\sigma_{1x}\sigma_{2y}$ and $\sigma_{1y}\sigma_{2x}$, the measurement of each is nonlocal. However, actually such operators can be measured locally [24]. We think that Mermin’s conclusion is right but it is because these two operators is not causally disconnected since each factor of one operator does not commute a factor of another operator.
- [19] M. Readhead, *Incompleteness, Nonlocality, and Realism*, (Clarendon, Oxford, 1987).
- [20] L. Vaidman, *Phys. Rev. Lett.* **70**, 3369 (1993).
- [21] I. Pitowsky, *Phys. Lett. A* **156**, 137 (1991); R. Clifton, C. Pagonis, and I. Pitowsky, in *Philosophy of Science Association 1992*, ed. D. Hull, M. Forbes, and K. Okruhlik (Philosophy of Science Association, East Kensing, 1992); R. Clifton and P. Niemann, *Phys. Lett. A* **166**, 177 (1992).
- [22] Vaidman’s work was cited as to be related in nonlocality arguments refuting the assertions against Lorentz invariance, see K. Berndl and S. Goldstein, *Phys. Rev. Lett.* **72**, 780 (1994), which was agreed by L. Hardy, *ibid.* **72**, 781 (1994).
- [23] Y. Aharonov and L. Vaidman, *J. Phys. A* **24**, 2315 (1991); Y. Aharonov, P. G. Bergmann, and J. L. Lebowitz, *Phys. Rev.* **134**, B1410 (1964).
- [24] See footnote 7 of [20] and Y. Aharonov, D. Z. Albert, and L. Vaidman, *Phys. Rev. D* **34**, 1805 (1986).
- [25] D. Bohm, *Phys. Rev.* **85**, 166 (1952); *ibid.* **85**, 180 (1952).